[Nyakinda*, 5.(1): January, 2015] ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

# IJESRT

## INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

# DETERMINISTIC MATHEMATICAL MODEL FOR FISH HARVESTING

**Joseph O. Nyakinda*, Omolo Ongati**

School of Mathematics and actuarial Jaramogi Oginga Odinga University of Science and Technology, Bondo, Kenya.

School of Mathematics and actuarial Jaramogi Oginga Odinga University of Science and Technology, Bondo, Kenya.

## ABSTRACT

Differential equations have been used to create mathematical models of real world systems in which rates of change are involved, for example in the study of how population grows or shrinks. One of the earliest models by Thomas Malthus has been found to be unrealistic since it predicts that population will grow exponentially and without bound – a prospect that defies physical limitations. Verhulst in his logistic population model developed a generalized version of the Malthusian model but added that natural factors such as disease and predators also affect mortality rate, and hence the rate of population growth. The Lokta – Voltera, which is a two species model, describes interactions between a predator and a prey in an ecosystem. The model forms the basis of many models used today in analysis of population dynamics, but unfortunately, in its original form, it lacks a stable equilibrium point. In mid 1980's Ragozin and Brown, established the existence of a steady state and described a unique approach to it for a predator Prey System.

Nile perch is a predator since it feeds on other small fish e.g. haplochromines, but it is also a prey because it is harvested for food. Rabuor and Polovina showed that there is a decline in the volume of Nile Perch and they attributed this to harvesting. So far, none of the models cited above considered harvesting as a factor affecting population growth. The need to develop a model for fish harvesting in general and in particular, the Lates Niloticus (Nile Perch) was Born out of this gap. The main objective of this study was to develop a model which can predict the amount of Nile perch harvested at any time t.

The procedure in this study involved using differential equations to yield specific insights into the management of complex ecological systems e.g. population outbreaks. We have analyzed the existing logistics model for equilibrium solutions and stability. To test on the stability of the model, we obtain secondary data from the Kenya Marine and Fisheries Institute (KMFRI) and Lake Basin Development Authority (LBDA). The data so obtained has been analysed using the concept of stabilityin order to develop the required model. We have managed in this thesis to construct three different models viz:

i) The Constant –rate harvesting model
ii) The Proportional – rate harvesting model
iii) A model depicting when harvesting is a function of P2(t) where P(t) represents the amount of Nile Perch harvested at any time t.

Stability analysis and verification of the models revealed that the proportional – rate harvesting model is more reliable as compared to the other two. This is because its solutions are closer to actual values of the Nile perch harvested for the period under investigation. It is hoped that the model would help economic and social planners in controlling the population of Lates Niloticus. The result will also give more insight in research in mathematical modeling.

## INTRODUCTION

The fisheries of lake Victoria have undergone dramatic changes since mid 1950s including: the introduction of alien species, increased fishing pressure with introduction of more efficient fishing gear and motorized fishing craft and

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

changes in the tropic status of the lake, Ochumba,Gophen and Pollinger[12]. The introduced Nile Perch, *Lates Niloticus*, preyed on the one abundant haplochromines, reducing their percentage contributions to catch the less than1% by weight, Graham[5].The catches in the Kenyan waters (Nyanza Gulf) of lake Victoria have therefore changed since the Nile Perch population explosion in early 1980s, Jackson[9].

The increase in Nile perch has been well documented, Okemwa[13]. Rabuor and Polivina[14], also showed the increase in landings upto 1991, which they attributed entirely to increasing fishing effort. They expressed concern about expanding fishery, stating that the fishery and stock were not in equilibrium and foresaw a substantial decline in catches. The scenario of increasing effort leading to increased catches deed indeed take place but the direct link to fishing effort ia an over-simplification, Ikiara[8]. Whilst fishing effort did increase between 1986 and 1989, it was stimulated by Nile perch stock at that time, Holling[7]. The concern of Rabuor and Polovina were well founded since the catches declined despite a continued increase in effort and by 1998 the catches were half those at the beginning of the decade, Getabu and Nyakundi[4].

To avoid a potential collapse in Nile perch Fishery, there is need to thoroughly asses the status of the stocks, thereby being able to predict future population of species. The main aim of this study was, therefore to develop a model to estimate the magnitude of Nile perch harvests at any time *t* in the Nyanza gulf of Lake Victoria for management purposes

## BASIC MATHEMATICAL CONCEPTS

**Theorem: Existence and uniqueness of solutions:[15]**

Suppose that a first- order ordinary differential equation can be written in the form

$$\frac{dP(t)}{dt} = f\left(t, P(t)\right), \qquad (2.1.1)$$

where both $f\left(t, p(t)\right)$ and its partial derivatives with respect to *t* and $P(t)$ are continuous in a rectangular region in the $t, P(t)$ -plane, $t_1 < t < t_2$, $P_1(t) < P(t) < P_2(t)$ (where any bounds maybe infinite). Then for any number $t_0$ and *a* within the region, there is an open interval containing $t_{0,}$ $a < t < b$, on which there exists precisely one solution of (2.1.1) satisfying the initial condition $P(t_0,) = a$ .For proof see[15]

**Definition of Boundedness**

A real valued function *f* is bounded on a region *R* if there is a constant *M* such that $\left| f(p) \leq M \right|$ for all *p* in *R*, Blanchard *et al* [2].

The precise sense of the word "region" does not matter. For the functions *f(x*) of one variable the region is usually an interval; for function *f(x ,y)*of two variables with *p* =(*x, y*) the region is usually a rectangular region in the plane , and similarly in higher dimensions. What is important is that the inequality $\left| f(p) \leq M \right|$ must hold for all values of the variable under consideration. As an example the function $f(x) = \frac{1}{x}$ is bounded for $1 \leq x \leq 2$, and also for $1 \leq x \leq \infty$, but not for $0 \leq x \leq 1$.

**Stability and Equilibrium solution [3]**

A solution *P*(*t*) of a differential equation of the form

$$\frac{dp(t)}{dt} = f\left(t, p(t)\right) \qquad 2.3.1$$

is said to be *stable* if for $\varepsilon > 0$ $(\varepsilon < \rho)$ there exists a positive number $\delta = \delta(\varepsilon)$ such that any solution $q(t)$ of (2.3.1) existing on interval *I* satisfies $\left|q(t) - p(t)\right| < \varepsilon, t \geq t_0$ whenever $\left|q(t_0) - p(t_0)\right| < \delta$ .

A solution $p(t)$ is said to be *asymptotically stable* if it is stable and if there exists a number $\delta_0 > 0$ such that any other solution $q(t)$ of (2.3.1), existing on interval $I$ is such that $|q(t)-p(t)| \to 0$ as $t \to \infty$ whenever $|q(t_0)-p(t_0)| < \delta_0$. A solution $p(t)$ is said to be *unstable* if it is not stable, Meyer [11].

For a differential equation of the form (2.3.1), the value $P^*$ is called an *equilibrium* level if $f(P^*) = 0$. We observe that if $P^*$ is in an equilibrium level, then the constant function $P(t) = P^*$ satisfies equation (2.3.1), and hence it is called an *equilibrium solution* of the differential equation (2.3.1).

**Example 3.1.** Consider the Logistic differential equation given by:

$$\frac{dp(t)}{dt} = r\left(1 - \frac{p(t)}{k}\right)p(t), \qquad 2.3.2$$

where $r$ is the intrinsic rate of growth and $k$ is the carrying capacity. The equilibrium levels are obtained by setting $\frac{dp(t)}{dt} = 0$, which then gives

$$r\left(1 - \frac{p(t)}{k}\right)p(t) = 0 \qquad 2.3.3$$

Solving for $p(t)$, we obtain the equilibrium solutions $p(t)$ =0 and $p(t)$= $k$.

**Definition 2.1**

A *neighborhood* of a number $S$ is any open interval (a,b) where $a < S < b$, Guterman and Nitecki[6]. An equilibrium level $P^*$ for the differential equation $\frac{dp(t)}{dt} = f\left(t, p(t)\right)$ is *stable* if there exists a neighborhood N of $P^*$ with the property that whenever $P_0 \in N$, then the solution $P(t)$, with the initial condition $P(t)$= $P_0$,

i) Is finite for all $t > t_0$

ii) Has $\lim_{t\to\infty} P(t) = P^*$

$N$ is called the *neighborhood of stability*

The Malthusian Equation is of the form

$$\frac{dp(t)}{dt} = rP(t), \qquad 2.4.1$$

for some constants of proportionality $r$ (the growth constant). Assuming $P(t) > 0$ (which is a reasonable assumption, since $P(t)$ represents a population), we have

$$\frac{1}{P(t)}\frac{dp(t)}{dt} = r \qquad 2.4.2$$

Integrating (2.4.), we obtain

$$P(t) = P(O)e^{rt}, \qquad 2.4.3$$

where $P_0 = P(0)$is the initial population size.

**Solution of the logistic differential equation.**

Assuming the rate of growth of Nile Perch Population follows the Verhulst's[16] logistic growth model,

$$\frac{dp(t)}{dt} = rP(t) - \frac{r}{k}p^2(t) \qquad 2.5.1$$

Where $r$ is the intrinsic growth rate coefficient

P(t) is the population size
*t* is the time period,
*k* is the carrying capacity (Is maximum limit beyond which population cannot be sustained)

Equation (2.5.1) can be simplified to

$$\frac{dp(t)}{dt} = rP(t)\left(1 - \frac{p(t)}{k}\right) \quad 2.5.2$$

Integrating both sides of equation (2.5.2) and simplifying we have

$$In\left(\frac{P(t)}{k - P(t)}\right) = rt + C, \quad 2.5.3$$

Where *C* is a constant.

To find *C*, we evaluate both sides of equation (2.5.3) at $t = 0$; $C = \ln \frac{P(0)}{k - P(0)}$, and after simplification, equation (2.5.3) becomes

$$P(t) = \frac{P(0)k}{P(0) + (k - P(0))e^{-rt}} \quad 2.5.4$$

From equation (2.5.4), as $t \to \infty$, $P(t) \to k$ (carrying capacity).

As $t \to 0, P(t) \to P(0)$.

## CONSTRUCTION OF LOGISTIC MODEL WITH HARVESTING

### Introduction

Consider a population *P*(*t*) of Nile Perch, with a growth rate *r*, in an environment with a carrying capacity *k*. We take a standard simple model of population growth, the logistic model. We then consider different ways of harvesting the fish, with the goal of maximizing long-term yield. We will attempt to solve the logistic equation with harvesting analytically and then analyze the situation for stability.

There are two standard approaches to harvesting from a population. We can harvest a set number of individuals every time (constant harvesting), or we can harvest a set percentage of population every time (proportional harvesting). The basic model of the *unharvested* population is given by (2.5.2)

$$\frac{dp(t)}{dt} = r\left(1 - \frac{p(t)}{k}\right)p(t), \quad (3.1.1)$$

If the harvesting rate is given by a nonnegative function $\psi(P(t))$ then the balance law yields the ordinary differential equation

$$\frac{dp(t)}{dt} = r\left(1 - \frac{p(t)}{k}\right)p(t) - \psi(P(t)), \quad (3.1.2)$$

Where *r* and *k* are positive constants.

If we harvest $\lambda$ units of fish every unit of time , we get the *constant harvesting model*

$$\frac{dp(t)}{dt} = r\left(1 - \frac{p(t)}{k}\right)p(t) - \lambda,$$

where $\lambda = \psi(P(t))$ is a positive constant.

This model assumes that a fixed number of fishing licenses have already been sold and that they do not restrict the number of fish a license holder may catch in a day. In this sense we cannot control the people, but we can still chose the type of fish we put in the lake.

 ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

In the other model, if we harvest a fraction *h* of the population every unit of time, we get the *proportional or variable rate-harvesting model*. From equation (3.1.2), if we let $\psi(P(t)) = hP(t)$ then

$$\frac{dp(t)}{dt} = r\left(1 - \frac{p(t)}{k}\right)p(t) - hP(t), \qquad (3.1.4)$$

Since the number of people fishing at any given time and the number of fish caught per person are not predictable, $hP(t)$ represents the harvesting effect on average. This means that we are more interested in the *long-term behavior of fish population.* Also we are interested in whether the fish population is sustainable, which is another long term consideration.

In real life, setting a quota on all harvesters and then counting the harvest can enforce constant harvesting. Proportional harvesting is often enforced by limiting the number of days that harvesting is permitted, with the assumption that in a fixed period of time it is only possible to catch a certain percentage of fish available. Another way to enforce proportional harvesting is to do a periodic census, and then adjust quota values for harvesters according to the current population figures.

**Basic assumptions of the study**

The study was based on the following assumptions

i) Each model developed in this study is a single species equilibrium model that assumes a constant environmental situation.. However, the Nile perch stock depends on several ecological conditions like food supply water, temperature, disease, pollution, currents and so on.

ii) The model further assumes that both the birth and death rates are the same for all intervals. Let the constant *d* be the death rate so that, for all $t \geq 0$

$$b = \frac{B(t)}{P(t)} \text{ and } d = \frac{D(t)}{P(t)}$$

iii) The models also assume that there is no migration into or out of the population, that is , the only source of population change is birth, death and harvesting.

**Population under harvesting**

Consider Verhulst logistic differential equation given by (3.1.1)

$$\frac{dP(t)}{dt} = r\left(1 - \frac{P(t)}{k}\right)P(t) \qquad (3.2.1)$$

Equation (3.2.1) describes population growth under environmental constraints. The harvesting model, as shown in the equation (3.1.2) is given as

$$\frac{dP(t)}{dt} = r\left(1 - \frac{P(t)}{k}\right)P(t) - \psi(P(t)) \qquad (3.2.2)$$

Where $\psi(P(t))$ is the harvesting component.

**Constant – rate harvesting model**

Suppose that $P(t)$ represents the population of Nile perch, and that fishing removes a certain number $\lambda$ of fish each unit of time. This meant there will be a term in $P(t + \Delta t) - P(t)$ equals to $-\lambda\Delta t$. When we divide by $\Delta t$ and take limits, we arrive at the equation for resources under constant harvesting:

$$\frac{dP(t)}{dt} = r\left(1 - \frac{P(t)}{k}\right)P(t) - \lambda, \qquad (3.2.3)$$

Where $\lambda = \psi(P(t))$ is a constant function.

Separating variables. we get

$$\frac{dP(t)}{-rP^2(t)+rkP(t)-\lambda k}=\frac{1}{k}dt \tag{3.2.4}$$

Integrating equation (3.2.4) and solving for $P(t)$ we get

$$P(t)=\frac{\left(\frac{P(0)+\beta}{\alpha-rP(0)}\right)\alpha-\beta e^{-\left(\frac{\alpha+r\beta}{k}\right)t}}{e^{-\left(\frac{\alpha+r\beta}{k}\right)t}+\left(\frac{P(0)+\beta}{\alpha-rP(0)}\right)r}, \tag{3.2.5}$$

Where $\alpha=\frac{rk\pm\sqrt{r^2k^2-4r\lambda k}}{2}$ and $\beta=\frac{rk\pm\sqrt{r^2k^2-4r\lambda k}}{2r}$

From equation (3.2.5)

As $t\to 0, P(t)\to P(0)$.

As $t\to\infty, P(t)\to\frac{\alpha}{r}$

When $P(t)\to\frac{\alpha}{r}$ and $\lambda=0$ (no harvesting) then

$P(t)=\frac{2rk}{2r}=k$ (carrying capacity).

**Proportional – rate harvesting Model**

Suppose that a certain proportion *h* of fish are caught per unit of time (the more fish the easier to catch). This means that instead of the term $-\lambda\Delta t$ for the number of fish taken away in an interval of length $\Delta t$, we would now have a term of the form $-hP(t)\Delta t$, which is proportional to the population. The differential equation that follows is

$$\frac{dP(t)}{dt}=r\left(1-\frac{P(t)}{k}\right)P(t)-h\big(P(t)\big) \tag{3.2.6}$$

Where *h* is the rate of harvesting.

Obtaining antiderivatives of both sides of equation (3.2.6), we have

$$\int\frac{dP(t)}{P(t)\big(\beta P(t)-\alpha\big)}=\int dt \tag{3.2.7}$$

Integrating equation (3.2.7) and solving for $P(t)$, we obtain

$$P(t)=\frac{\alpha P(0)}{\beta P(0)+\big(\alpha-\beta P(0)\big)e^{-\alpha t}}, \tag{3.2.8}$$

Where $\alpha=r-h$ and $\beta=\frac{r}{k}$.

From equation (3.2.8), as $t\to 0, P(t)\to P(0)$ and

As $t\to\infty, P(t)\to\frac{\alpha}{\beta}$ or $P(t)=\frac{k(r-h)}{r}$.

If *h*=0 (no harvesting), then $P(t)=k$ (carrying capacity).

**When harvesting is a function of** $P^2(t)\left(\psi(P(t)) = hP^2(t)\right)$.

If the proportion of fish caught per unit of time depends on the power $P^2(t)$, then the harvesting equation becomes

$$\frac{dP(t)}{dt} = r\left(1 - \frac{P(t)}{k}\right)P(t) - h\left(P^2(t)\right) \qquad (3.2.9)$$

Separating the variables in equation (3.2.9)

$$\frac{dP(t)}{P(t)\left(mP(t) - r\right)} = -dt \qquad (3.2.10)$$

Solving to $P(t)$ we get $P(t) = \dfrac{-rP(0)}{p(t)\left(mP(0) - r\right)e^{-rt} - m}$, (3.2.11)

Where $m = \dfrac{r}{k} + h$.

From equation (3.2.11)

As $t \to 0, P(t) \to \dfrac{-rP(0)}{\left(mP(0) - r\right) - m}$. As $t \to \infty, P(t) \to \dfrac{r}{\frac{r}{k} + h} = \dfrac{kr}{r + kh}$

When $h$=0 (no harvesting), $P(t) \to k$ (carrying capacity)

We observe that the harvesting model (equation (3.2.3)) has unrealistic behavior close to $P(t) = 0$: the differential equation (3.2.3) implies that at $P(t) = 0$, it yields

$$\frac{dP(t)}{dt} = -\lambda \qquad (3.2.12)$$

This implies that $P(t)$ continues to decrease, and would become negative. This is meaningless because population cannot be negative. According to Holing [7], a way to fix this defect in the modelis to assume that the harvesting equation

$$\frac{dP(t)}{dt} = r\left(1 - \frac{P(t)}{k}\right)P(t) - h\frac{\lambda P(t)}{\lambda + P(t)} \qquad (3.2.13)$$

Where h is the rate of harvesting and $\lambda$ is the quantity harvested.

Equation (3.2.13) has equilibrium at $P(t) = 0$ for all parameters, and the rate at which fish are caught decreases with $P(t)$.This is practicable when fewer fish are available, it is harder to find them and so the daily catch drops. Also when there are sufficiently manyfish (when P(t) is large ), $\lim\limits_{P(t)\to\infty} h\dfrac{\lambda P(t)}{\lambda + P(t)} = h$ the fish level is close to $h$

Expanding and simplifying equation (3.2.13)

$$\frac{dP(t)}{dt} = \frac{rk\lambda P(t) + rkP^2(t) - r\lambda P^2(t) - rp^3(t) - hk\lambda P(t)}{k(\lambda + P(t))} \qquad (3.2.14)$$

Separating the variables we have

$$\frac{(\lambda + P(t))dP(t)}{P(t)\left\{-rP^2(t) + mP(t) + n\right\}} = \frac{1}{k}dt \qquad (3.2.15)$$

Evaluating equation (3.2.15) using partial fractions we obtain

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

$$\frac{(\lambda+P(t))dP(t)}{P(t)\left\{-rP^{2}(t)+mP(t)+n\right\}}=\frac{\lambda}{nP(t)}+\frac{\frac{\lambda r}{n}P(t)+\frac{n-\lambda m}{n}}{-rP^{2}(t)+mP(t)+n}=\frac{1}{k}dt \qquad (3.2.16)$$

Integrating both sides of equation (3.2.16), we have

$$\int\frac{(\lambda+P(t))dP(t)}{P(t)\left\{-rP^{2}(t)+mP(t)+n\right\}}=\frac{\lambda}{n}\int\frac{dP(t)}{P(t)}+\int\frac{\lambda rP(t)+(n-\lambda m)}{-rP^{2}(t)+mP(t)+n}dP(t)=\frac{1}{k}\int dt \qquad (3.2.17)$$

$$\frac{\lambda}{n}\ln P(t)-\frac{\lambda}{2n}\ln\left(P^{2}(t)-\frac{m}{r}P(t)-\frac{n}{r}\right)-\frac{\lambda r}{n\sqrt{m^{2}+4rn}}\arctan\left(\frac{2r}{\sqrt{m^{2}+4rn}}\left(P(t)-\frac{m}{2r}\right)\right)=\frac{1}{k}t+c \qquad (3.2.18)$$

From equation (3.2.18),

When $P(t)=\frac{m}{2r},\left(P(t)-\frac{m}{2r}\right)\to 0$ and $P(t)\to 0$. (Extinction).

When $P(t)<\frac{m}{2r},\left(P(t)-\frac{m}{2r}\right)\to "-ve"$ this does not make sense since population cannot be negative.

When $P(t)>\frac{m}{2r},\left(P(t)-\frac{m}{2r}\right)\to "+ve"$ and $P(t)\to k$. (carrying capacity).

**Verification of the model**

**Data Parameter estimates**

The following table gives an illustration on the catch time series.

***Table 1:Annual catches of Nile Perch, lates niloticus, from the Nyanza Gulf of lake Victoria , 1979 – 1998***

| Year | Catches in tonnes | Percentage composition |
|---|---|---|
| 1979 | 4,286,000 | 14.0120 |
| 1980 | 4,310, 000 | 16.0158 |
| 1981 | 22,835, 000 | 59.8099 |
| 1982 | 33,134, 000 | 54.3555 |
| 1983 | 52,37, 000 | 67.9004 |
| 1984 | 41,319, 000 | 57.4993 |
| 1985 | 50,029, 000 | 56.4732 |
| 1986 | 64,929, 000 | 55.2442 |
| 1987 | 86,833, 000 | 60.4176 |
| 1988 | 82,020, 000 | 48.9402 |
| 1989 | 119,276, 000 | 41.9534 |
| 1990 | 118,504, 000 | 38.4275 |
| 1991 | 122,781, 000 | 28.4061 |
| 1992 | 105,980, 000 | 51.1388 |
| 1993 | 109,196, | 52.1284 |
| 1994 | 88,838, 000 | 53.7020 |
| 1995 | 102,427, 000 | 56.3127 |
| 1996 | 96,472, 000 | 57.9545 |
| 1997 | 73,005, 000 | 37.2163 |
| 1998 | 76,664, 000 | 40.0322 |

From table 1, and using MATLAB, we obtain the following parameter estimates:
$h$ = 0.170341985
$k$ = 345,036, 000
$r = 0.41728643 \pm 0.1$
And we let $P(0)$ = 22,835,000 tonnes (1981 harvest

**Constant – rate harvesting model**
From table 1

For the period 1990 – 1991,
$r$ =0.41728643, $k$ =345,036,000, $P$ (0) = 22,835,000 and
$0 < \lambda \leq 35{,}994{,}710.17$
For the maximum value of $\lambda$
$\alpha$ =71,989,420.33 and $\beta$ =172,518,000
The population in 1991 would be :
$P$(1991) = 168,492,057 tonnes.

For the period 1995 – 1996,
$r$ =0.31728643, $k$ =345,036,000, $P$ (0) = 22,835,000 and
$0 < \lambda \leq 27{,}368{,}810.17$
For the maximum value of $\lambda$
$\alpha$ =54737620.33 and $\beta$ =172,518,000
The population in 1996 would be :
$P$(1996) = 167,194,647.8 tonnes.

**Proportional – rate harvesting model**
For the period 1990 – 1991,
$r$ =0.41728643, $h$ =0.1703441985, $k$ =345,036,000, $P$ (0) = 22,835,000 and
The population in 1991 would be :
$P$(1991) = 122,11,901.8 tonnes.

For the period 1995 – 1996,
$r$ = 0.31728643, $h$ =0.1703441985, $k$ =345,036,000, $P$ (0) = 22,835,000 and
The population in 1996 would be :
$P$(1996) = 96,157,373.31 tonnes.

**When harvesting is a function of $P^2$(t)**
For the period 1995 – 1996,
$r$ = -0.31728643, $h$ =0.1703441985, $k$ =345,036,000, $P$ (0) = 22,835,000 and
The population in 1996 would be :
$P$(1996) = 0.015965907 tonnes.

This formula is only applicable when $r < 0$, since for $r > 0$, $P(t) < 0$, which is meaningless.

**Remarks**
From the above example, it follows that the proportional-rate harvesting model is more reliable as compared to the other two models developed. This is because its solutions are closer to the value in the table as compared to the results obtained from the other models. The reliability of the model could be enhanced by obtaining more accurate data of

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785

Nile Perch population. However, we had noted earlier on that in Kenya , data collection and compilation is still a problem.

## CONCLUSION

In this paper, we have managed to develop the following models;

i) The constant – rate harvesting model
ii) The proportional – rate harvesting model
iii) We have also considered a case where fish population depends on the power $P^2(t)$.

Stability analysis and verification of the models revealed that proportional - rate harvesting model is more reliable as compared to the other two models developed.

The accuracy of the catch - time series data is always affected by several factors not least, the way it is gathered. Therefore the data spanning the period1981 – 1998 have been used because it includes a period when the percentage composition of Nile Perch in the total catch was in excess of 60%. It also represents a period of Nile Perch domination of fish harvests from Lake Victoria. Dealing with data in this period therefore should help to improve the quality of results.

## REFERENCE

[1] Asila A.A. Working document for Lake Victoria Fisheries Managementdecision, Draft No. 3, KMFRI Kenya, March 2001.
[2] Blanchard P, Devoney L.R and Hall R.G, Preliminary Edition – differential equations, Brooks/Cole Publishing Company, ITP, U.S.A, 1999.
[3] Deo S.G. and Rahavendra V,Ordinary Differential Equations and stability theory, TATA McGraw-Hill publishing company Ltd, New Delhi, 1980.
[4] Getabu A. and Nyaundi, J. Relative abundance and distribution of fish in Kenyan waters of Lake Victoria, FIDAWOG workshop, Kisumu, 1999
[5] Graham, M. The Victoria Nyanza and its Fisheries. A report on fishing survey of Lake Victoria 1927 – 1928, London Crown Agents ,41pp, 1929.
[6] Guterman M.M. and Nitecki H. Z. Differential equations – a first course, saunders college publishing, Third Edition U.S.A. 1991.
[7] Holling C.S. Some characteristics of simple types of predation and parasitism, Canadian Entomologist 91,PP. 385, 1959.
[8] Ikiara M. and Bokea C. The Macro economy of the export fishing industry in Kenya side of Lake Victoria report number 7 April 2000.
[9] Jackson P. B. N. the African Great Lakes: Past , Present and Future. African Journal of Tropical Hydrobiology and Fisheries 1,35 – 49, 1971.
[10] Malthus T. “An essay on the principle of population”, Anthony Flew, ed Penguin books, Baltimore, 1978.
[11] Meyer J.W. “Concepts of Mathematical modelling”. McGraw -hill Book Company, United states 1976.
[12] Ochumba P.B.O, Gophen, M and Pollinger, U. Ecological changes in Lake Victoria after the invasion by Nile Perch (Lates Niloticus): the catchment, water quality and fisheries, Mombasa Kenya, KMFRI, pp 29-39, 1994.
[13] Okemwa, E. N. Changes in fish species composition of Nyanza Gulf of Lake Victoria. KMFRI workshop on Aquatic Resources of Kenya, 13-19, July 1981, Mombasa pp 138-156, 1981.
[14] Rabuor, C.O. and Polovina, J. J. An analysis of the multigear, multispecies fishery in the Kenyan waters of Lake Victoria. NAGA, The ICLARM Quarterly, april 1995, pp 34 -37, 1995.
[15] Redhefer R. and Port D. Introduction to differential equations, Jones and Bartlett Publishers, London U.S.A. 1992.
[16] Verhulst, F. P. Generating Extra – terrestrial Terrain, Vol.15, No 2 Belgium, 1845.